UC San Diego

Lecture 18:

Map Network and Cycle Consistency

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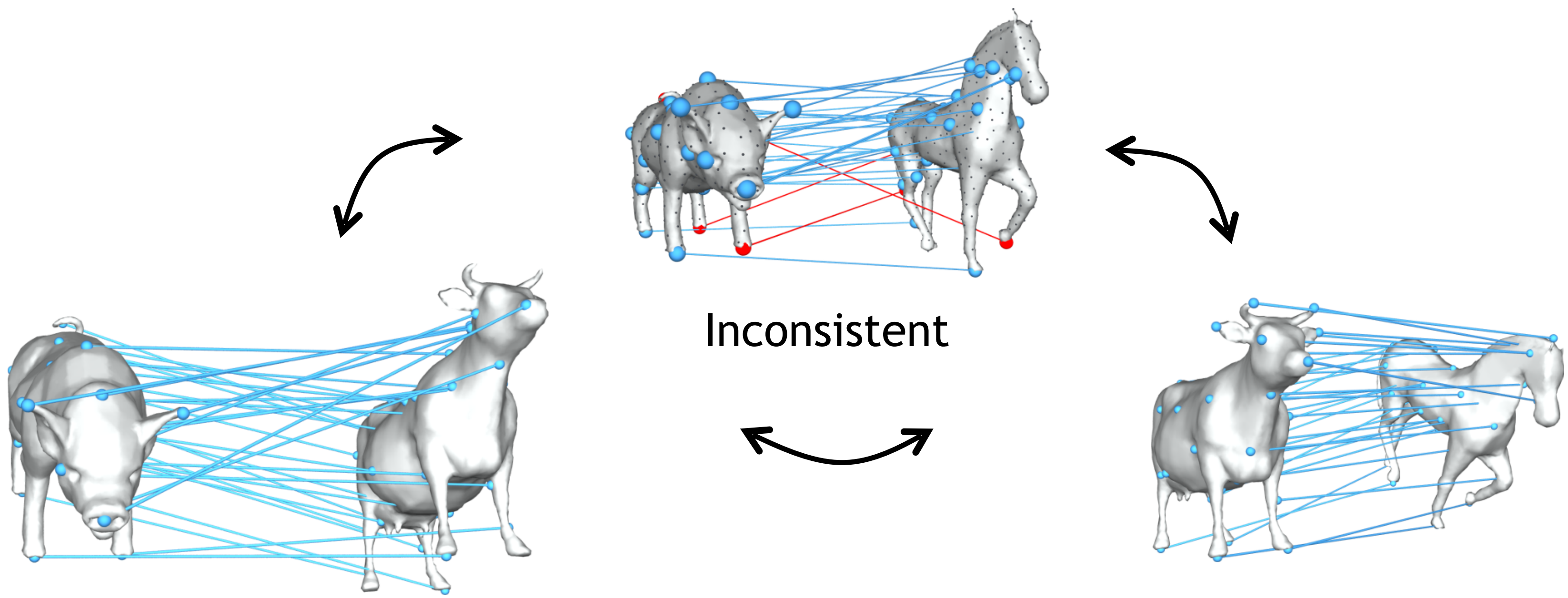
Mar 13, 2018

Slides ack: Qixing Huang

A natural constraint on maps is that they should be consistent along cycles



Q. Huang, G. Zhang, L. Gao, S. Hu, A. Bustcher, and L. Guibas. *An Optimization Approach for Extracting and Encoding Consistent Maps in a Shape Collection*, SIGGRAPHAsia' 12

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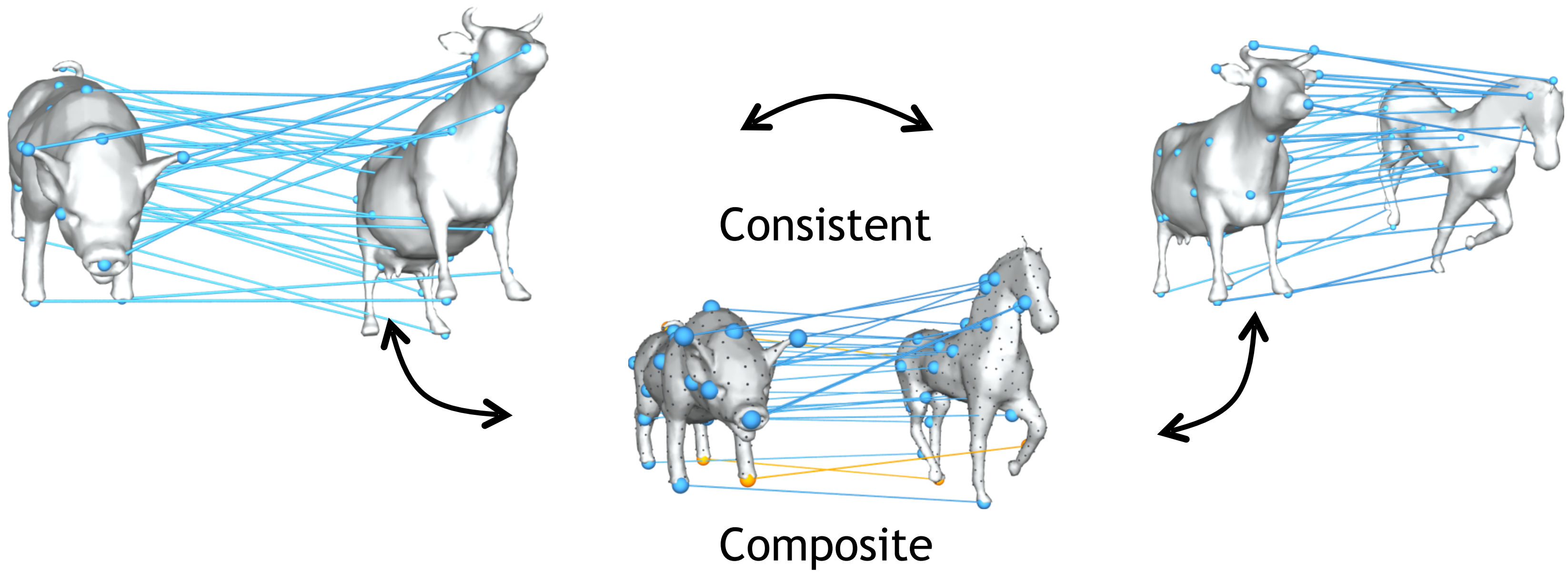

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Literature on leveraging cycle-consistency for map synchronization

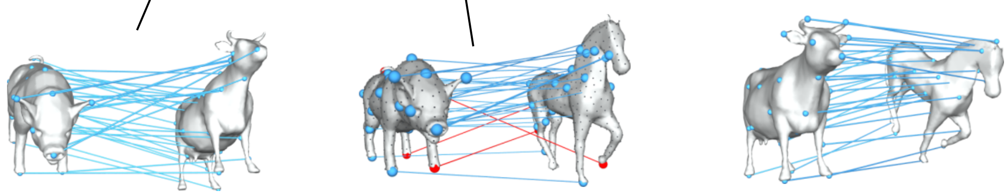
- Spanning tree optimization [Huber et al. 01, Huang et al. 02]

- Sampling inconsistent cycles [Zach et al. 10, Nyugen et al. 11, Zhou et al. 15]

- MRF formulation [Cho et al. 08, Crandel et al. 11, Huang et al. 12]

Map synchronization as constrained matrix optimization

$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{21} & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & I_m \end{bmatrix}$$



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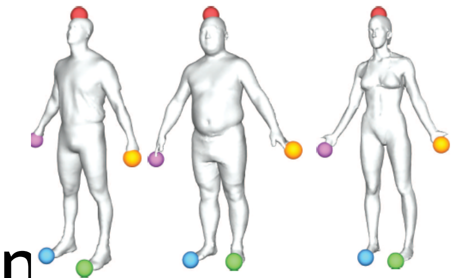
Noisy measurements of matrix blocks

Q. Huang and L. Guibas, *Consistent Shape Maps via Semidefinite Programming*, Sym. on Geometry Processing'13
Y. Chen, L. Guibas, Q. Huang, *Near-Optimal Joint Object Matching via Convex Relaxation*, ICML'14
Q. Huang, F. Wang, L. Guibas, *Functional map networks for analyzing and exploring large shape collections*, SIGGRAPH'14

Algorithms

Permutation synchronization



- Input:
  - n objects, each object has m poin
  - p2p maps along an object graph

$$\mathcal{G} = (\mathcal{S}, \mathcal{E})$$

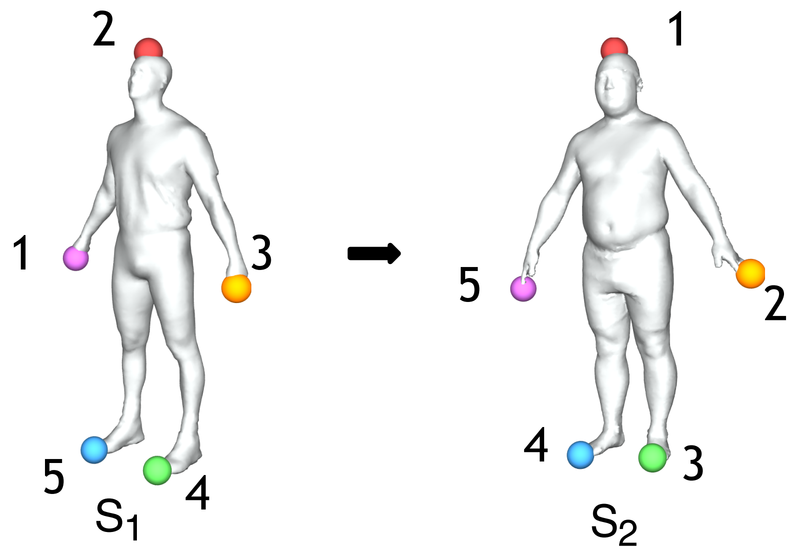
- Output: one-to-one maps between all pairs of objects

$$\Phi = \{\phi_{ij} : S_i \to S_j | 1 \leq i, j \leq n\}$$

  - Cycle-consistent
  - Close to the input ma

$$\begin{aligned}
\phi_{ii} &= id_{S_i}, \quad 1 \leq i \leq n, && \text{(1-cycle)}\\
\phi_{ji} \circ \phi_{ij} &= id_{S_i}, \quad 1 \leq i < j \leq n, && \text{(2-cycle)}\\
\phi_{ki} \circ \phi_{jk} \circ \phi_{ij} &= id_{S_i}, \quad 1 \leq i < j < k \leq n, && \text{(3-cycle)}
\end{aligned}$$

Matrix representation of maps

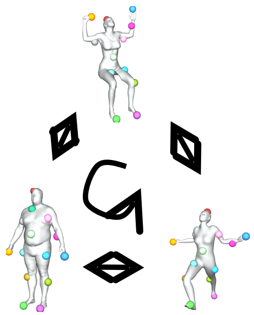


$$X_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & X_{nn} \end{bmatrix}$$

- ➢ Diagonal blocks are identity matrices
- ➢ Symmetric
- ➢ Off diagonal blocks are permutation matrices

The equivalence between cycle-consistency and positive semidefiniteness



Cycle-consistent

$$X = \begin{bmatrix} I_m & X_{12} & \cdots & X_{1n} \\ X_{21} & I_m & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & I_m \end{bmatrix} \succeq 0$$

$$\begin{aligned} \phi_{ii} &= id_{S_i} \\ \phi_{ji} \circ \phi_{ij} &= id_{S_i} \\ \phi_{ki} \circ \phi_{jk} \circ \phi_{ij} &= id_{S_i} \end{aligned}$$

$$X = \begin{bmatrix} I_m \\ \vdots \\ X_{n1} \end{bmatrix} \begin{bmatrix} I_m & \cdots & X_{n1} \end{bmatrix}$$

SDP => Cycle-consistency

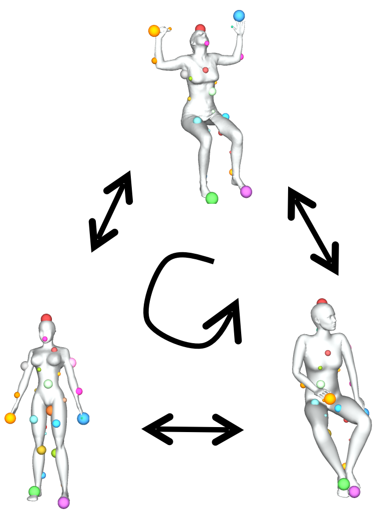
$$\boldsymbol{D} = \mathrm{Diag}(\boldsymbol{I}_m, \boldsymbol{X}_{12}, \cdots, \boldsymbol{X}_{1n}) \in \mathbb{R}^{nm \times nm}$$

$$\boldsymbol{X}' = \boldsymbol{D}\boldsymbol{X}\boldsymbol{D}^T = \begin{pmatrix} \boldsymbol{I}_m & \boldsymbol{I}_m & \cdots & \boldsymbol{I}_m \\ \boldsymbol{I}_m & \ddots & \boldsymbol{X}_{1i}\boldsymbol{X}_{ij}\boldsymbol{X}_{1j}^T & \vdots \\ \vdots & \boldsymbol{X}_{1j}\boldsymbol{X}_{ij}^T\boldsymbol{X}_{1i}^T & \ddots & \vdots \\ \boldsymbol{I}_m & \cdots & \cdots & \boldsymbol{I}_m \end{pmatrix} \succeq 0$$

$$A_{33}(X'_{ij}(s,s)) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & X'_{ij}(s,s) \\ 1 & X'_{ij}(s,s) & 1 \end{pmatrix} \succeq 0 \Rightarrow \boldsymbol{X}_{ij} = \boldsymbol{X}_{1i}^T\boldsymbol{X}_{1j}$$

indices: (*s*, *im*+*s*, *jm*+*s*)

Parametrizing cycle-consistent maps



Cycle-consistent

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & X_{nn} \end{bmatrix}$$

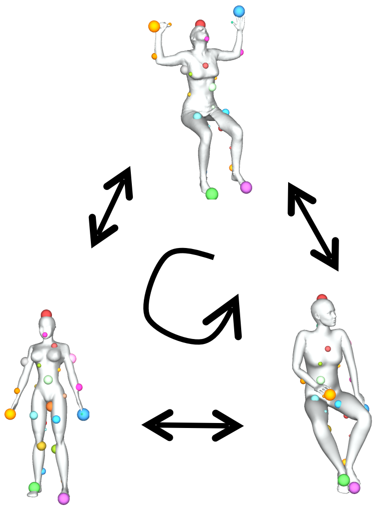
$$X_{ii} = I_m, \quad 1 \le i \le n$$

$$X_{ij}\mathbf{1} = \mathbf{1}, X_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \le i < j \le n$$

$$X \in \{0,1\}^{nm \times nm}$$

$$X \succeq 0$$

Relaxing the permutation constraints for convexity



Cycle-consistent

$$X = \begin{bmatrix} X_{11} & X_{12} & \cdots & X_{1n} \\ X_{21} & X_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & X_{n-1,n} \\ X_{n1} & \cdots & X_{n,n-1} & X_{nn} \end{bmatrix}$$

$$X_{ii} = I_m, \quad 1 \leq i \leq n$$

$$X_{ij}\mathbf{1} = \mathbf{1}, X_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \leq i < j \leq n$$

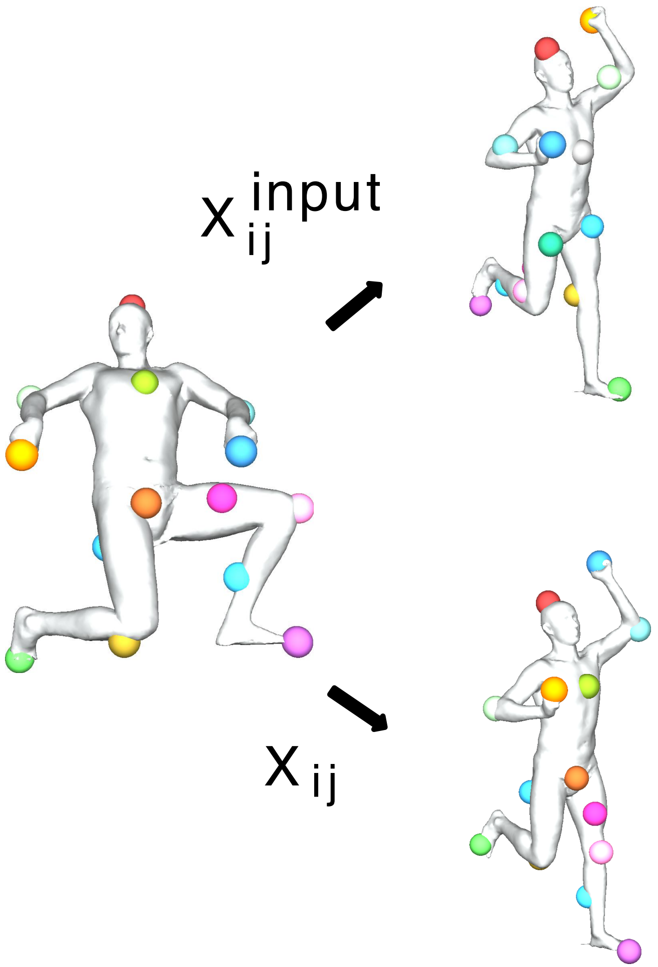
$$0 \leq X \leq 1$$

$$X \succeq 0$$

Tight relaxation!

The convex hull of permutation matrices

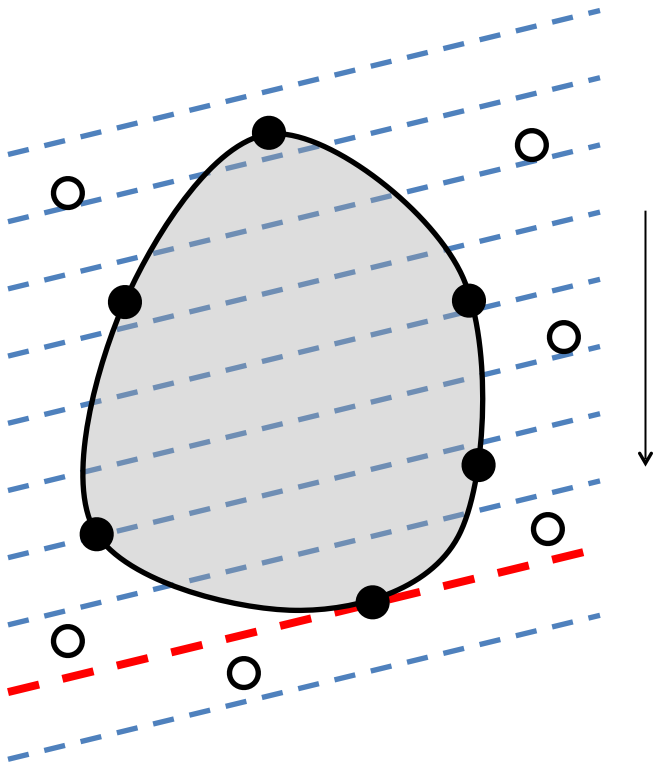
Objective Function



L1-norm:

$$f_{\text{align}} = \sum_{(i,j)\in\mathcal{G}} \|X_{ij}^{\text{input}} - X_{ij}\|_1$$

Semidefinite programming relaxation for permutation synchronization



$$\underset{X}{\text{minimize}} \sum_{(i,j)\in\mathcal{G}} \langle \mathbf{1}\mathbf{1}^T - 2X_{ij}^{\text{input}}, X_{ij}\rangle$$

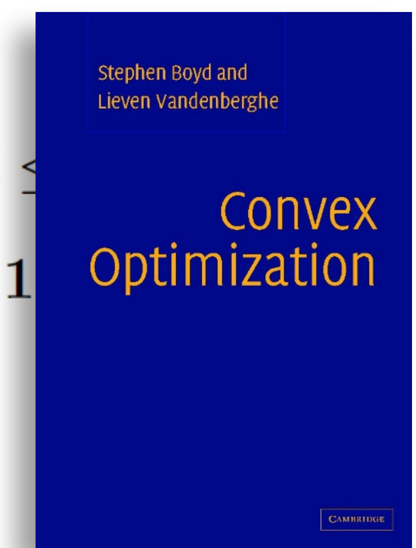
subject to

$$X_{ii} = I_m, \quad 1 \le i \le$$

$$X_{ij}\mathbf{1} = \mathbf{1}, X_{ij}^T\mathbf{1} = \mathbf{1} \quad n$$

$$0 \le X \le 1$$

$$X \succeq 0$$

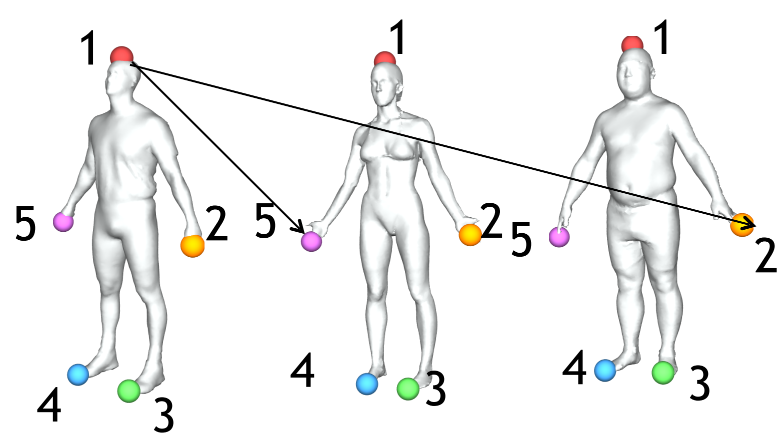


https://github.com/huangqx/CSP_Codes

Deterministic guarantee

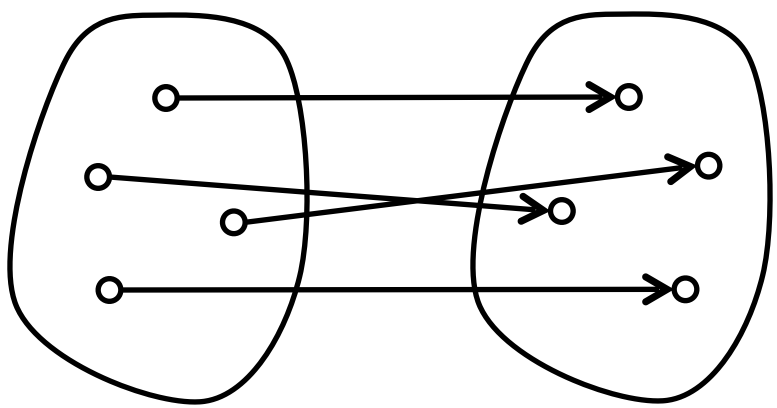
- *Theorem: Suppose the input maps are noisy perturbations of some underlying ground-truth maps. Then we can recover the underlying maps if*

  #incorrect corres. of each point $\frac{\lambda_2(G)}{4}$



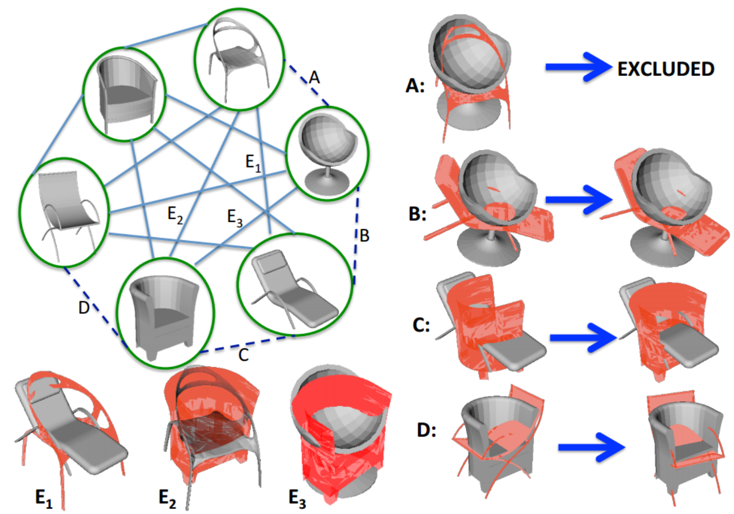
Optimality when the object graph G is a clique

- 25% incorrect correspondences
- Worst-case scenario
  - Two clusters of objects of equal size
  - Wrong correspondences between objects of different clusters only (50%)



Justification of maximizing $\lambda_2(G)$ for map graph construction

Imageweb [Heath et al 10]



Fuzzy correspondences on shapes [Kim et al 12]

Randomized setting

- Generalized Erdős–Rényi model:
  - p_{obs}: the probability that two objects connect
  - p_{true}: the probability that a pair-wise map is correct
  - Incorrect maps are random permutations

- Theorem [CGH'14]: *The underlying permutations can be recovered w.h.p if*

$$p_{\text{true}} \geq c\frac{\log^2(mn)}{\sqrt{np_{\text{obs}}}}$$

Optimality when m is a constant

- Exact recovery condition:

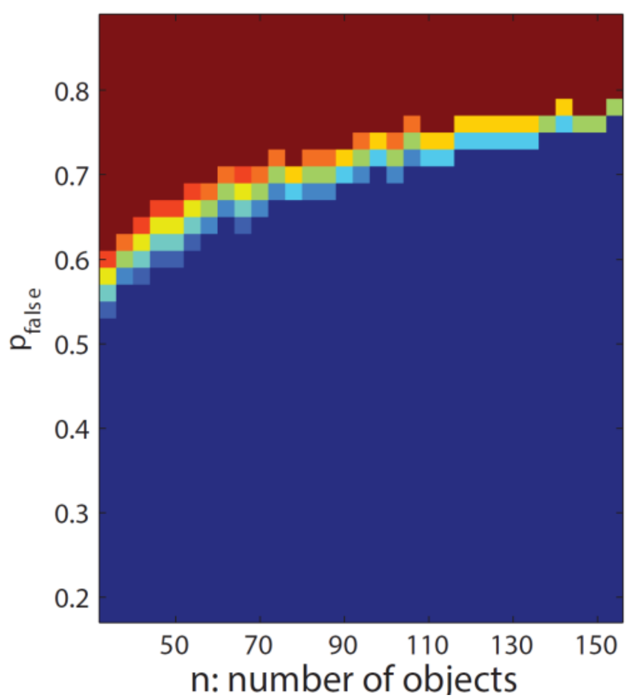
$$p_{\mathrm{true}} > c\frac{\log^2(n)}{\sqrt{np_{\mathrm{obs}}}}$$

- Information theoretic limits [Chen et al 15]:

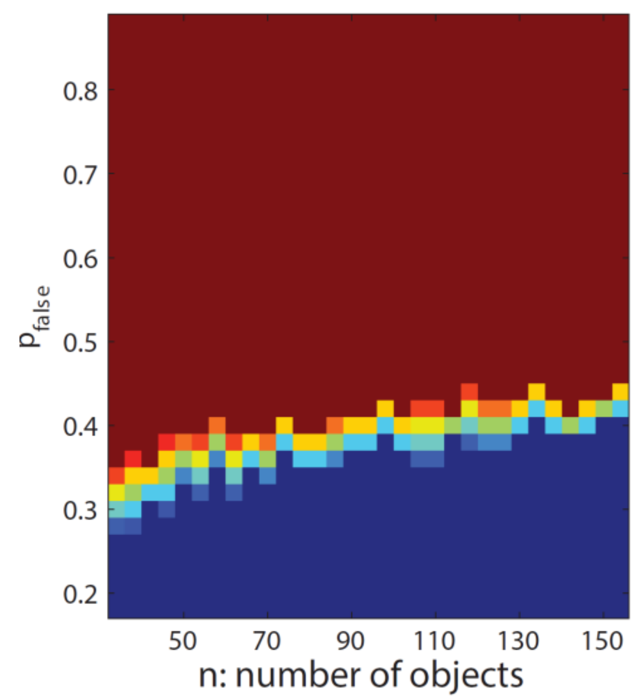
No method works if $p_{\mathrm{true}} \leq c_1\dfrac{1}{\sqrt{np_{\mathrm{obs}}}}$

Comparison to a generic low-rank matrix recovery method

Permutation synchronization



RPCA



Phase transitions in empirical success probability (p_{obs} =1)

Random-sign condition breaks when perturbing permutations

- RPCA can handle dense corruption if the perturbations exhibit random sign pattern, yet

$$E_{\mathcal{P}_m}\big(\mathrm{sgn}\big(X_{ij} - I_m\big)\big) = -I_m + \frac{1}{m}\mathbf{1}\mathbf{1}^T$$

- The map constraints incur a quotient space defined by

$$\mathcal{K} = \{Z : |Z \in \mathbb{R}^{m\times m},\ Z\mathbf{1} = 0,\ Z^T\mathbf{1} = 0\}$$

- The expectation under this quotient space

$$E_{\mathcal{P}_m/\mathcal{K}}\big(\mathrm{sgn}\big(X_{ij} - I_m\big)\big) = 0$$

How to handle partial maps?

$$
\begin{aligned}
\underset{X}{\text{minimize}} \quad & \sum_{(i,j)\in\mathcal{G}} \langle \mathbf{1}\mathbf{1}^T - 2X_{ij}^{\text{input}}, X_{ij} \rangle \\
\text{subject to} \quad & X_{ii} = I_m, \quad 1 \le i \le n \\
& X_{ij}\mathbf{1} = \mathbf{1}, X_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \le i < j \le n \\
& 0 \le X \le 1 \\
& X \succeq 0
\end{aligned}
$$

Reformulation

$$\underset{X}{\text{minimize}} \quad \sum_{(i,j)\in\mathcal{G}} \langle \mathbf{1}\mathbf{1}^T - 2X_{ij}^{\text{input}}, X_{ij} \rangle$$

$$\text{subject to} \quad X_{ii} = I_m, \quad 1 \le i \le n$$

$$X_{ij}\mathbf{1} = \mathbf{1}, X_{ij}^T\mathbf{1} = \mathbf{1}, \quad 1 \le i < j \le n$$

$$0 \le X \le 1$$

$$X \succeq 0$$

$$\begin{bmatrix} \cdots & \mathbf{1}^T \\ \mathbf{1} & X \end{bmatrix} \succeq 0$$

Q. Huang, Y. Chen, and L.Guibas, Scalable Semidefinite Relaxation for Maximum A Posterior Estimation, ICML’ 14

Partial point-based map synchronization

Step I: Spectral method:

m <= #dominant eigenvalues of X^{input} after trimmin

Step II:

$$\begin{aligned} \underset{X}{\text{minimize}} \quad & \sum_{(i,j)\in\mathcal{G}} \langle \lambda \mathbf{1}\mathbf{1}^T - 2X_{ij}^{\text{input}}, X_{ij} \rangle \\ \text{subject to} \quad & X_{ii} = I_{m_i}, \quad 1 \le i \le n \\ & 0 \le X \le 1 \\ & \begin{bmatrix} m & \mathbf{1}^T \\ \mathbf{1} & X \end{bmatrix} \succeq 0 \end{aligned}$$

Size of the universe

Exact recovery condition

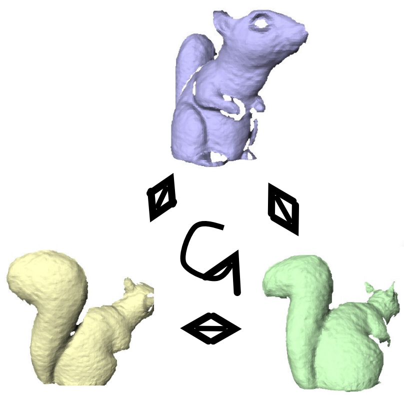
- Randomized model: *n* objects, universe size *m*
  - Each object contains a fraction p_{set} of m elements
  - Each pair is observed w.p. p_{obs}
  - Each observed is randomly corrupted w.p. $1 - p_{true}$

$$\lambda \in [\frac{1}{m}, \frac{1}{\sqrt{p_{obs}}}]$$

- Theorem. When , the underlying maps can be r gh probability if

$$p_{true} \geq c_2 \frac{\log^2(mn)}{p_{\mathrm{set}}^2 \sqrt{n p_{\mathrm{obs}}}}$$

Rotation synchronization



Cycle-consistent

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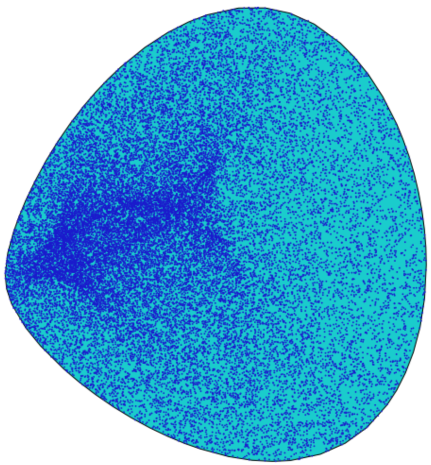
The equivalence also holds for rotations

What is the convex hull of SO(3)?

http://www.mit.edu/~parrilo/pubs/talkfiles/FoCM.pdf

$$\begin{bmatrix} Z_{11}+Z_{22}-Z_{33}-Z_{44} & 2Z_{23}-2Z_{14} & 2Z_{24}+2Z_{13} \\ 2Z_{23}+2Z_{14} & Z_{11}-Z_{22}+Z_{33}-Z_{44} & 2Z_{34}-2Z_{12} \\ 2Z_{24}-2Z_{13} & 2Z_{34}+2Z_{12} & Z_{11}-Z_{22}-Z_{33}+Z_{44} \end{bmatrix}, \quad Z \succeq 0, \quad \operatorname{Tr} Z = 1.$$

The dimension of the convex hull is 9



Robust rotation synchronization [Wang and Singer 13]

- Formulation:

$$\begin{aligned} \underset{X}{\text{minimize}} \quad & \sum_{(i,j)\in\mathcal{G}} \|X_{ij} - X_{ij}^{\text{input}}\|_{\mathcal{F}} \\ \text{subject to} \quad & X_{ii} = I_m, \quad 1 \le i \le m \\ & X \succeq 0 \end{aligned}$$

- Exact recovery condition [Wang and Singer' 13]:

$$p_{\text{true}} \ge (1 - p(m))$$

$$p_c\,(2) \approx 0.4570,\ p_c\,(3) \approx 0.4912,\ \ \textit{and}\ p_c\,(4) \approx 0.5186$$

Ongoing effort on non-convex optimization (SO(3))

- Initial solution via connection Laplacian

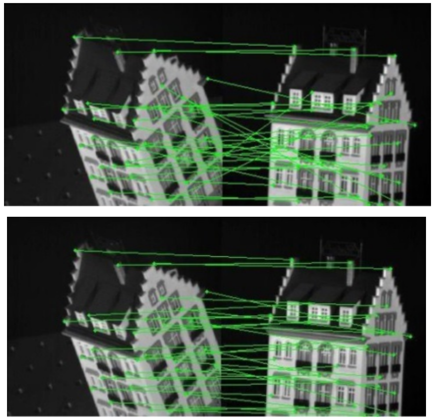
$$L = L^{\mathcal{G}}_{\text{inlier}} \otimes I_3 + L_{\text{noise}}$$

- Refine the solution via gradient descent of reweighted least squares:

$$\begin{aligned} \underset{\{R_i\}}{\text{minimize}} \quad & \sum_{(i,j)\in\mathcal{G}} \|R_j - R_{ij}^{\text{init}} R_i\| \\ \text{subject to} \quad & R_i \in SO(3) \end{aligned}$$

Applications

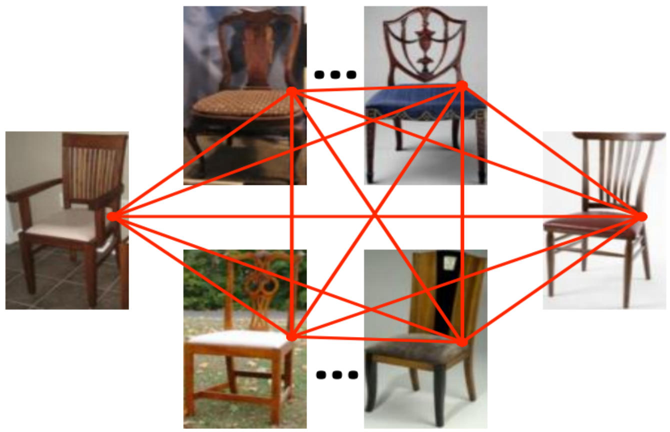
Map synchronization versus learning pair-wise matching



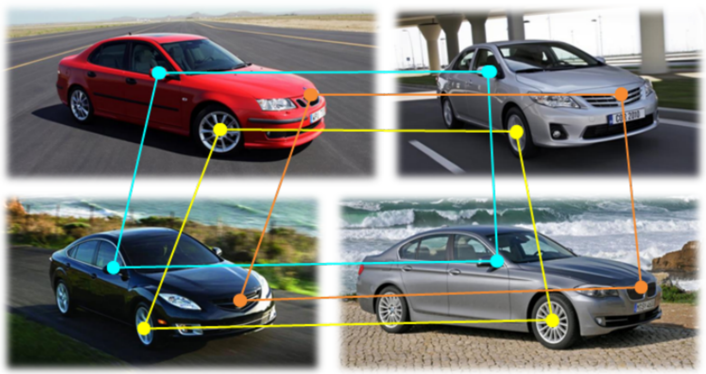
CMU Hotel dataset

Pair-wise (RANSAC)	Pairwise (Learning) Leordeanu et al. 12	Joint Matching (from RANSAC)
64.1%	94.8%	99.9%

Follow-up works at CVPR/ICCV

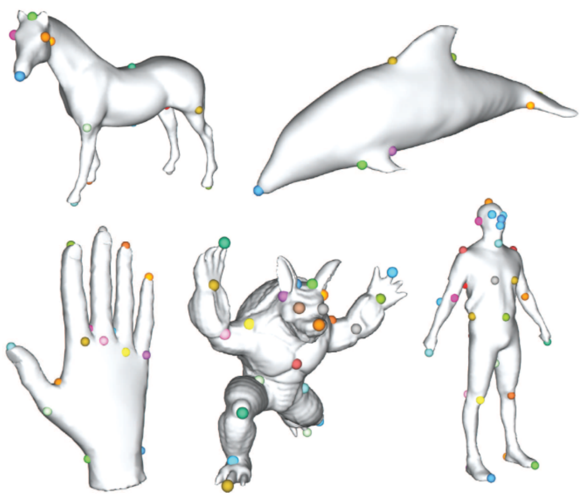


Flowweb [Zhou-Lee-Yu-Efros 15]

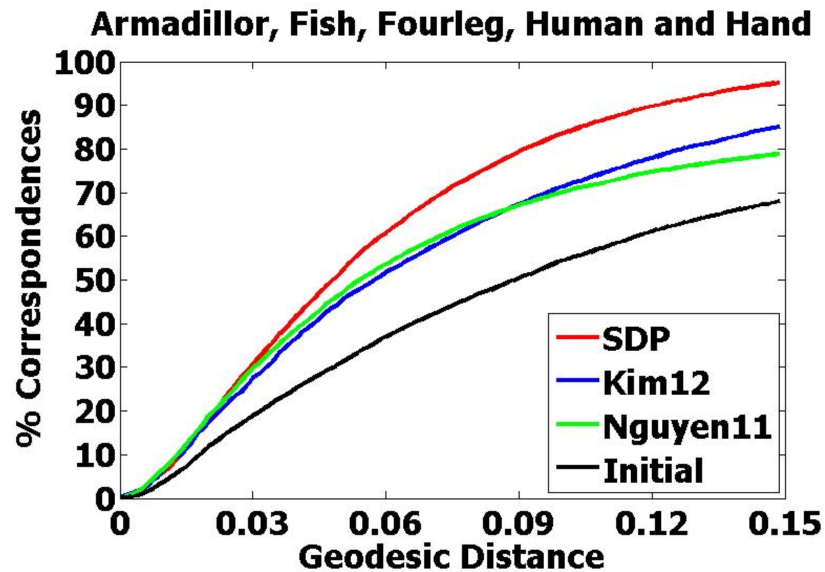


Fast Alternating Minimization
[Zhou-Zhu-Daniilidis 15]

Similar behavior on establishing shape correspondences



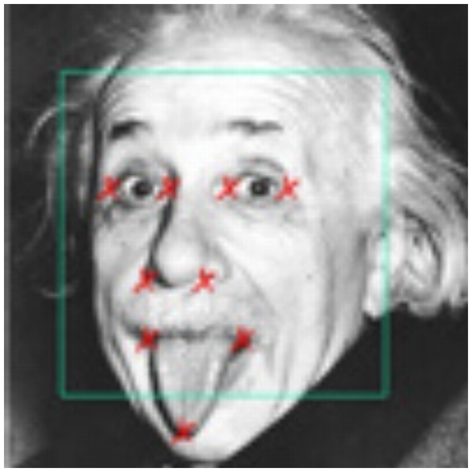
SHREC07-UnSym

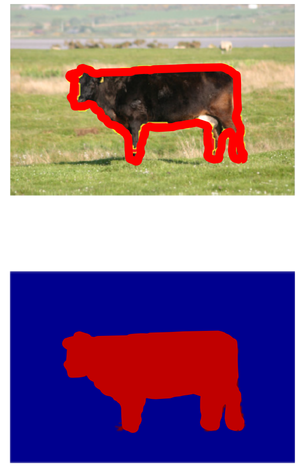


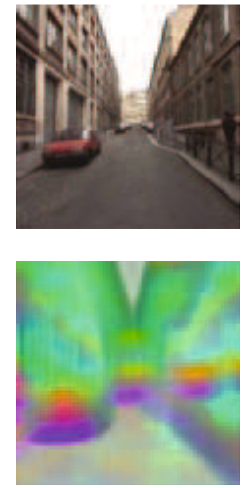
How to find relations among objects exhibit significant variabilities




The functional representation [Ovsjanikov et al. 12]







Point features

Segments

Descriptors

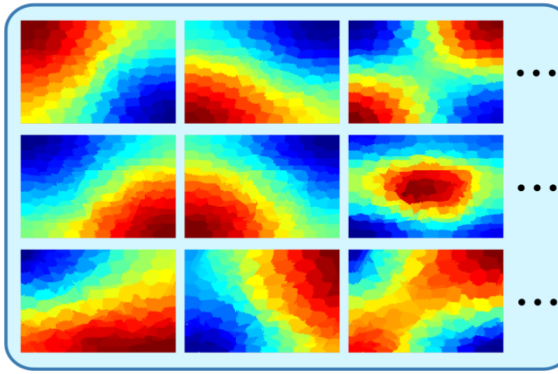
Delta functions

Indicator functions

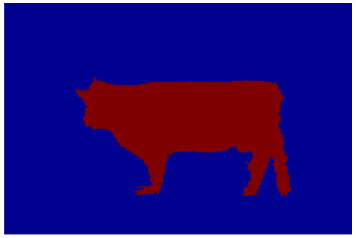
The space of functions is linear (dim = #pixels)

Reduced functional space

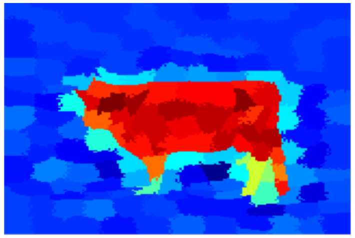
- Basis of functional space
  - First M Laplacian eigenfunctions of a graph of super-pixels



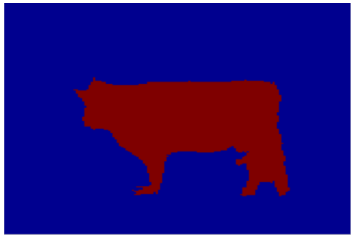
- Reconstruct any function with small error (M=30)



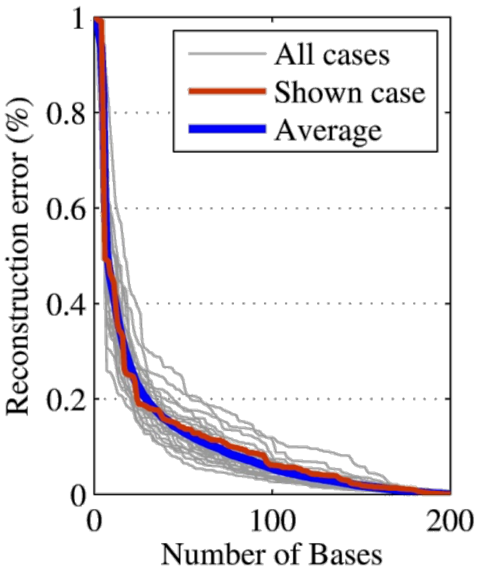
Binary indicator function



Reconstructed function

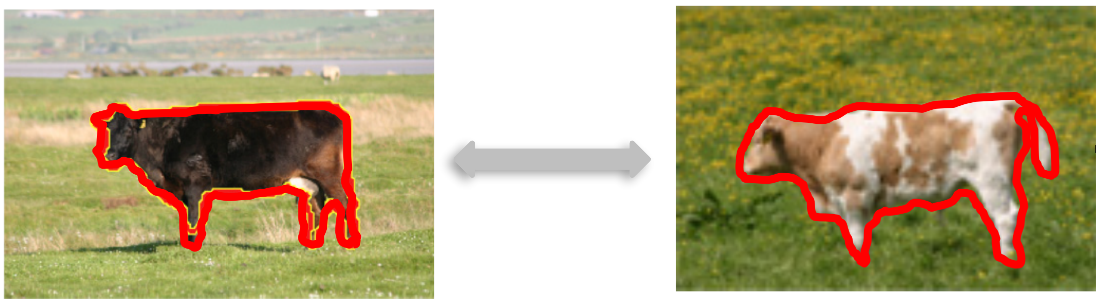


Thresholded reconstructed function

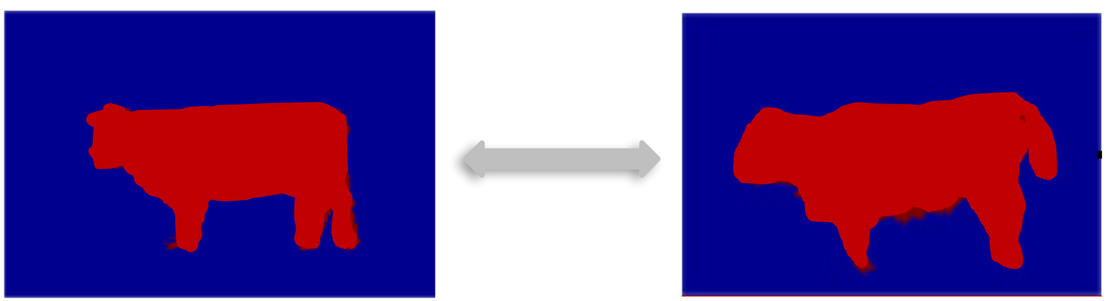


Reconstruction error

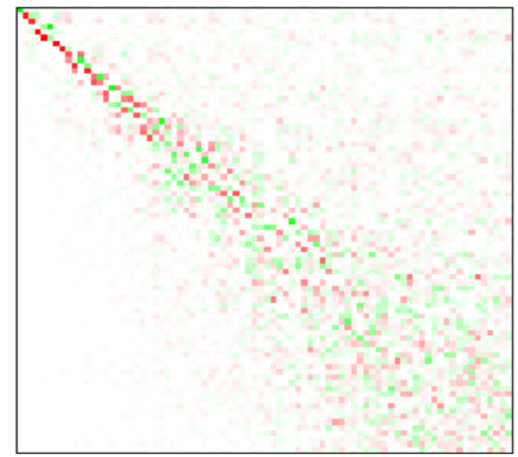
Functional map representation



Segmentation correspondence



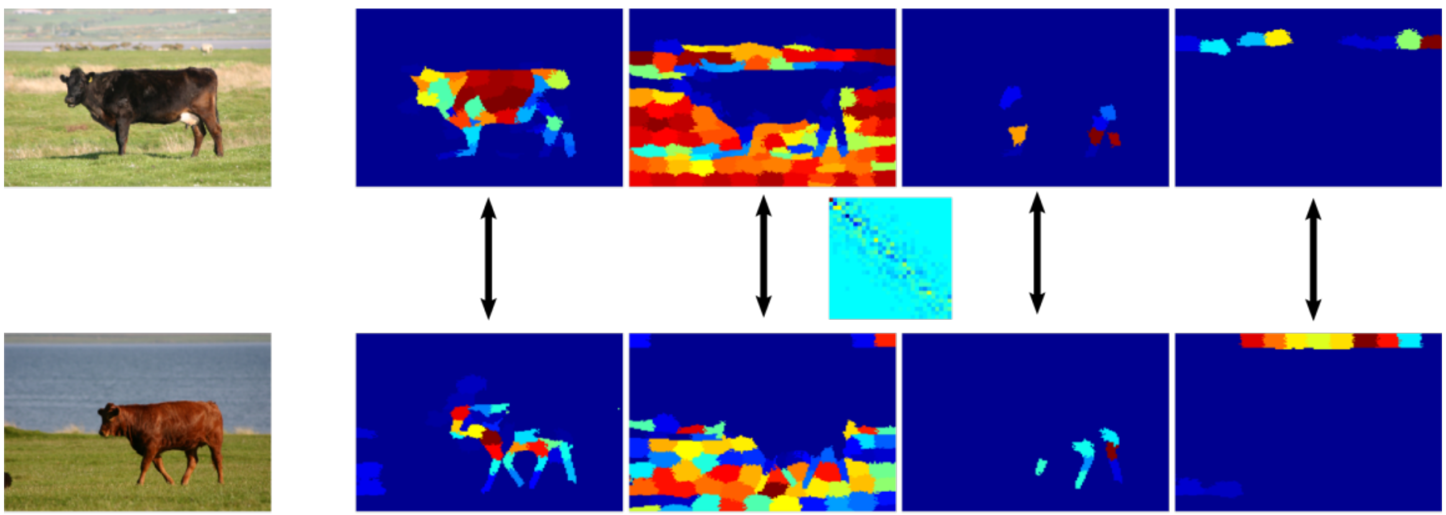
function correspondence



Functional map

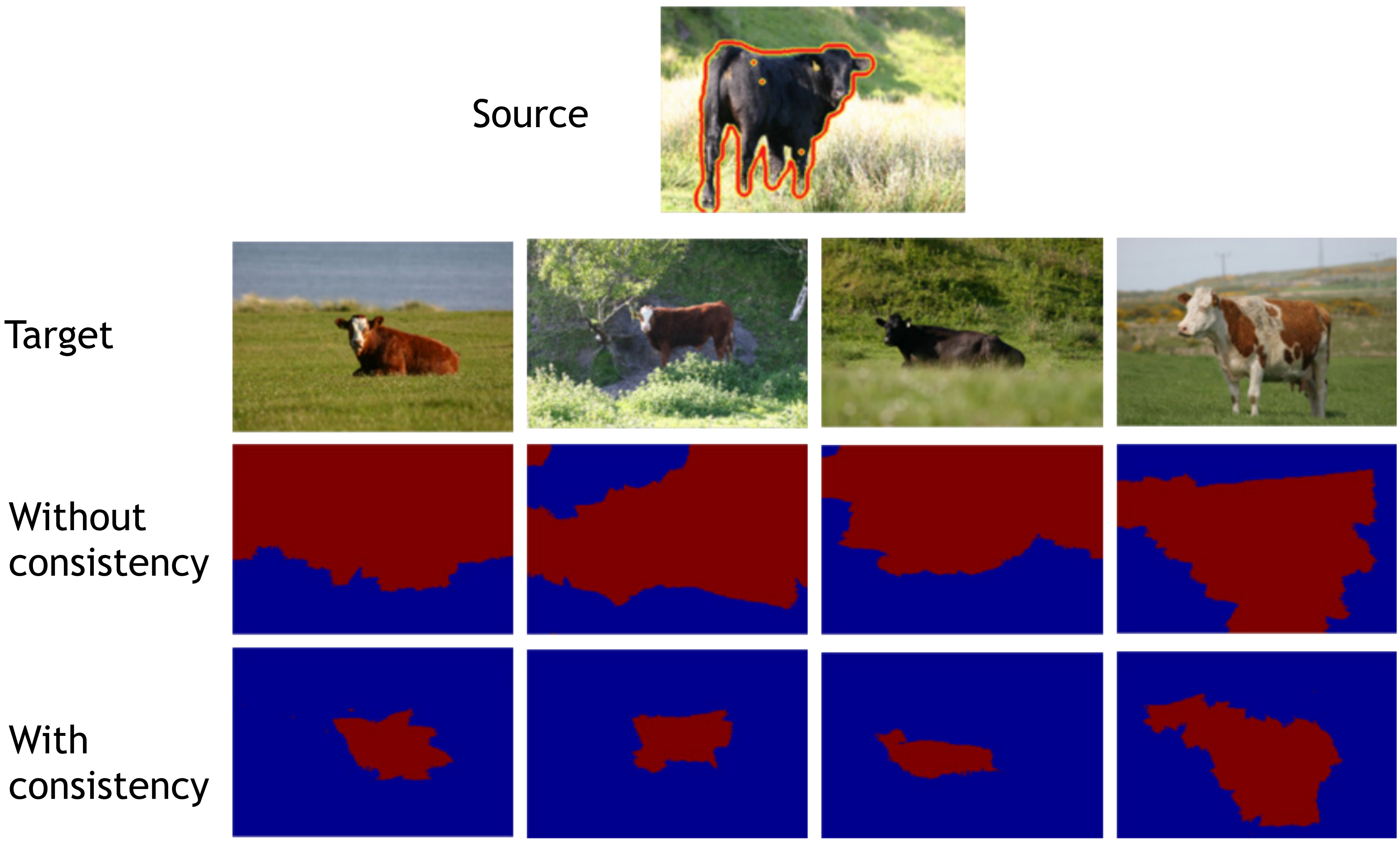
Functional map computation

- Can be computed from point correspondences and/or descriptors by solving a linear system



$$X^\star = \arg\min_X \sum \|X\boldsymbol{f}_i - \boldsymbol{g}_i\|_F^2$$

The effect of enforcing consistency on functional maps



Joint image segmentation as consistent normalized-cuts

- Two objectives for optimizing segmentations
  - Aligning edge features

$$f^{\text{seg}} = \sum_i \boldsymbol{f}_i^T L_i^{\text{NCut}} \boldsymbol{f}_i \quad s.t. \ \|\boldsymbol{f}_i\| = 1$$

  - Segmentation consistency

$$f^{\text{map}} = \sum_{(i,j)\in\mathcal{G}} \|X\boldsymbol{f}_i - \boldsymbol{f}_j\|^2$$

- Joint optimization:

$$\text{minimize } f^{\text{seg}} + \gamma f^{\text{map}} \quad s.t. \ \sum_i \|\boldsymbol{f}_i\|^2 = 1$$

- iCoseg data set
- New unsupervised method
  - Mostly outperforms other unsupervised methods
  - Sometimes even outperforms supervised methods
  - Supervised input is easily added and further improves the results

Kuettel'12 (Supervised)		Unsupervised Fmaps
Image+transfer	Full model	
87.6	91.4	90.5

Supervised method

Class	Joulin '10	Rubio '12	Vicente '11	Fmaps -uns
Alaska Bear	74.8	86.4	90.0	**90.4**
Red Sox Players	73.0	90.5	90.9	**94.2**
Stonehenge1	56.6	87.3	63.3	**92.5**
Stonehenge2	86.0	88.4	88.8	87.2
Liverpool FC	76.4	82.6	87.5	**89.4**
Ferrari	85.0	84.3	89.9	**95.6**
Taj Mahal	73.7	88.7	91.1	**92.6**
Elephants	70.1	75.0	43.1	**86.7**
Pandas	84.0	60.0	92.7	88.6
Kite	87.0	89.8	90.3	**93.9**
Kite panda	73.2	78.3	90.2	**93.1**
Gymnastics	90.9	87.1	91.7	90.4
Skating	82.1	76.8	77.5	78.7
Hot Balloons	85.2	89.0	90.1	**90.4**
Liberty Statue	90.6	91.6	93.8	**96.8**
Brown Bear	74.0	80.4	95.3	88.1
Average	78.9	83.5	85.4	**90.5**

MSRC

Unsupervised performance comparison

Class	N	Joulin '10	Rubio '12	Fmaps -uns
Cow	30	81.6	80.1	**89.7**
Plane	30	73.8	77.0	**87.3**
Face	30	84.3	76.3	**89.3**
Cat	24	74.4	77.1	**88.3**
Car(front)	6	87.6	65.9	87.3
Car(back)	6	85.1	52.4	**92.7**
Bike	30	63.3	62.4	**74.8**

Supervised performance comparison

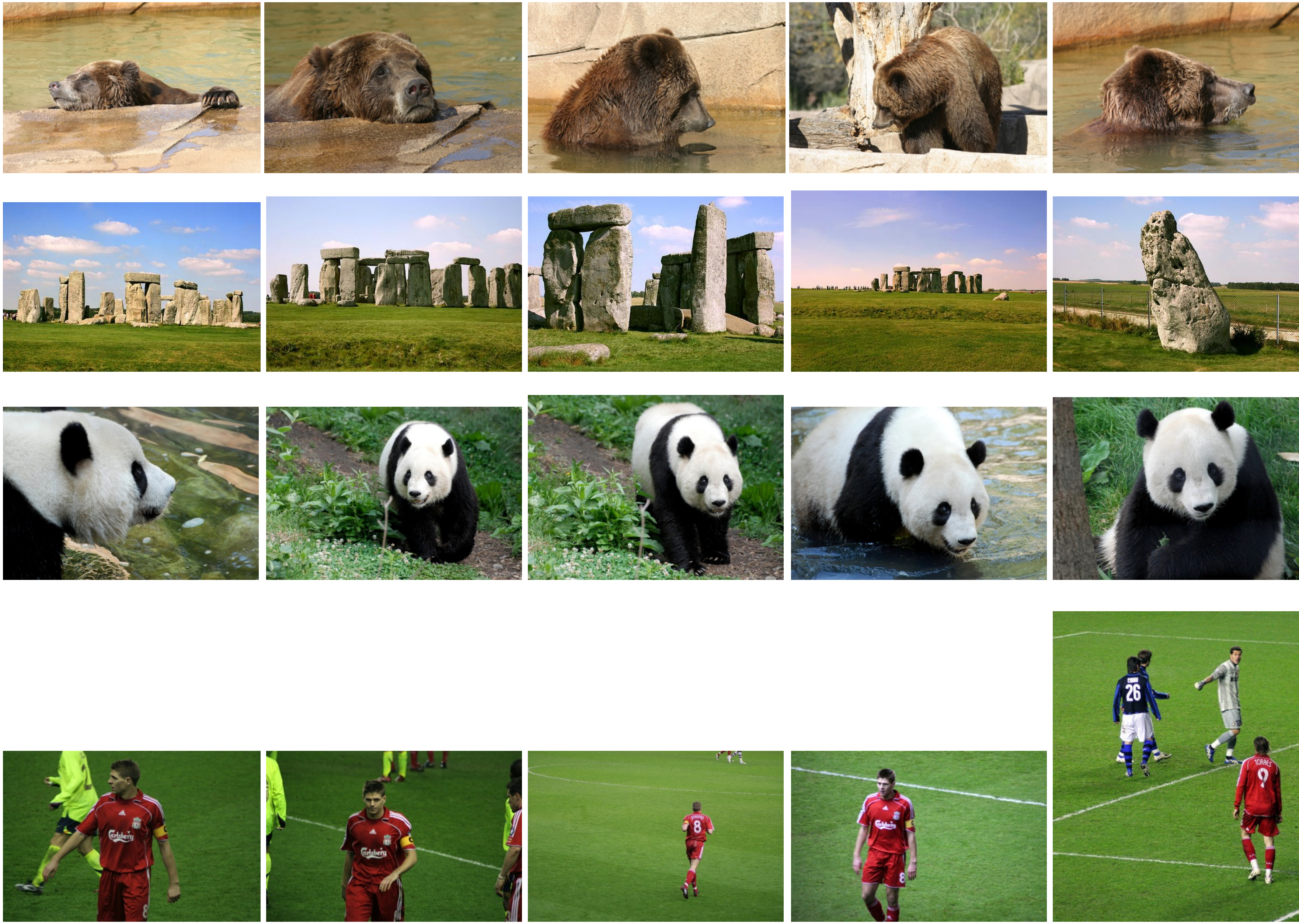
Class	Vicente '11	Kuettel '12	Fmaps -s
Cow	94.2	92.5	**94.3**
Plane	83.0	86.5	**91.0**
Car	79.6	88.8	83.1
Sheep	94.0	91.8	**95.6**
Bird	95.3	93.4	**95.8**
Cat	92.3	92.6	**94.5**
Dog	93.0	87.8	91.3

• PASCAL

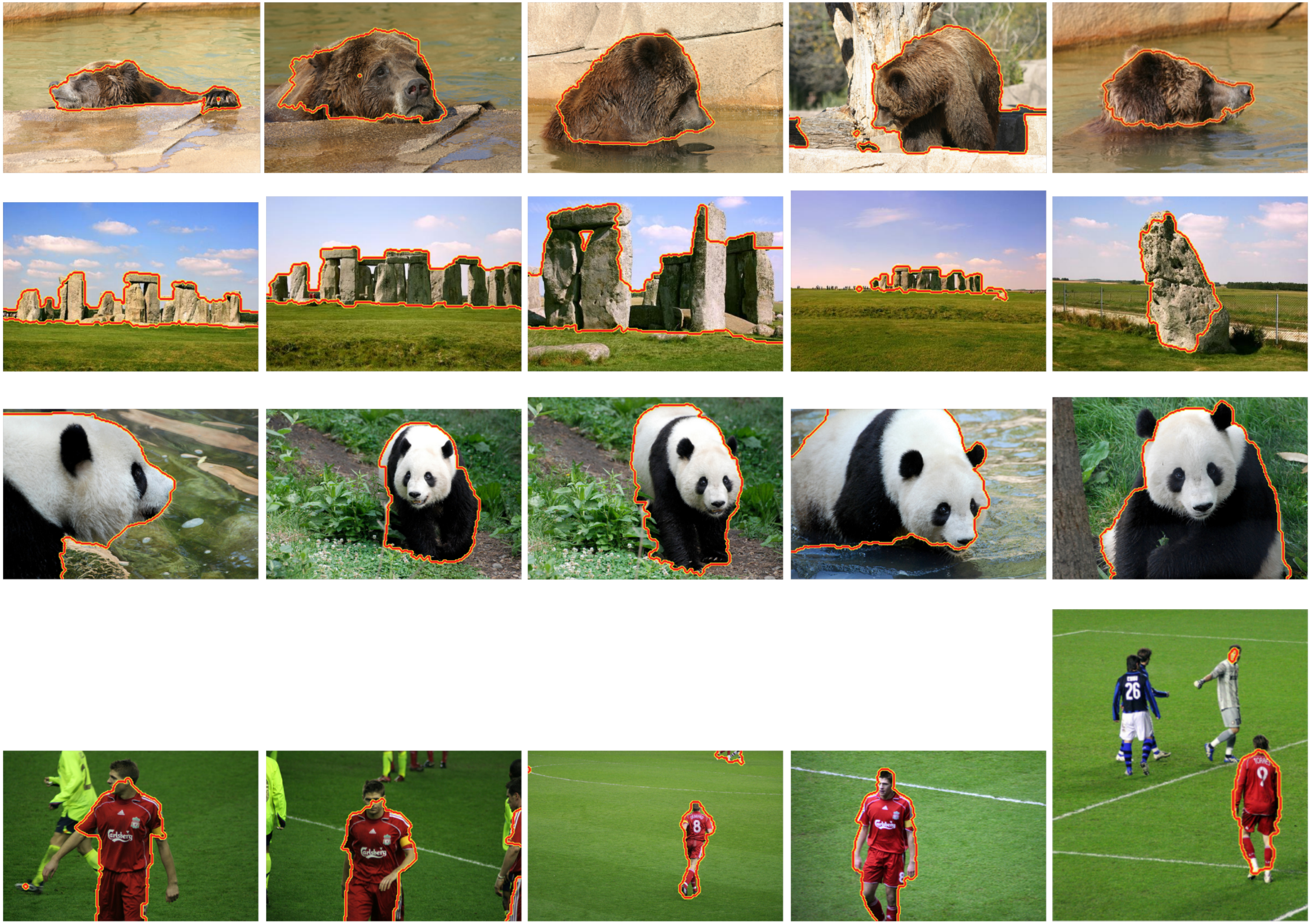
Class	N	L	Kuettel '12	Fmaps -s	Fmaps -uns
Plane	178	88	90.7	**92.1**	89.4
Bus	152	78	81.6	**87.1**	80.7
Car	255	128	76.1	**90.9**	82.3
Cat	250	131	77.7	**85.5**	82.5
Cow	135	64	82.5	**87.7**	85.5
Dog	249	121	81.9	**88.5**	84.2
Horse	147	68	83.1	**88.9**	87.0
Sheep	120	63	83.9	**89.6**	86.5

- New method mostly outperforms the state-of-the-art techniques in both supervised and unsupervised settings

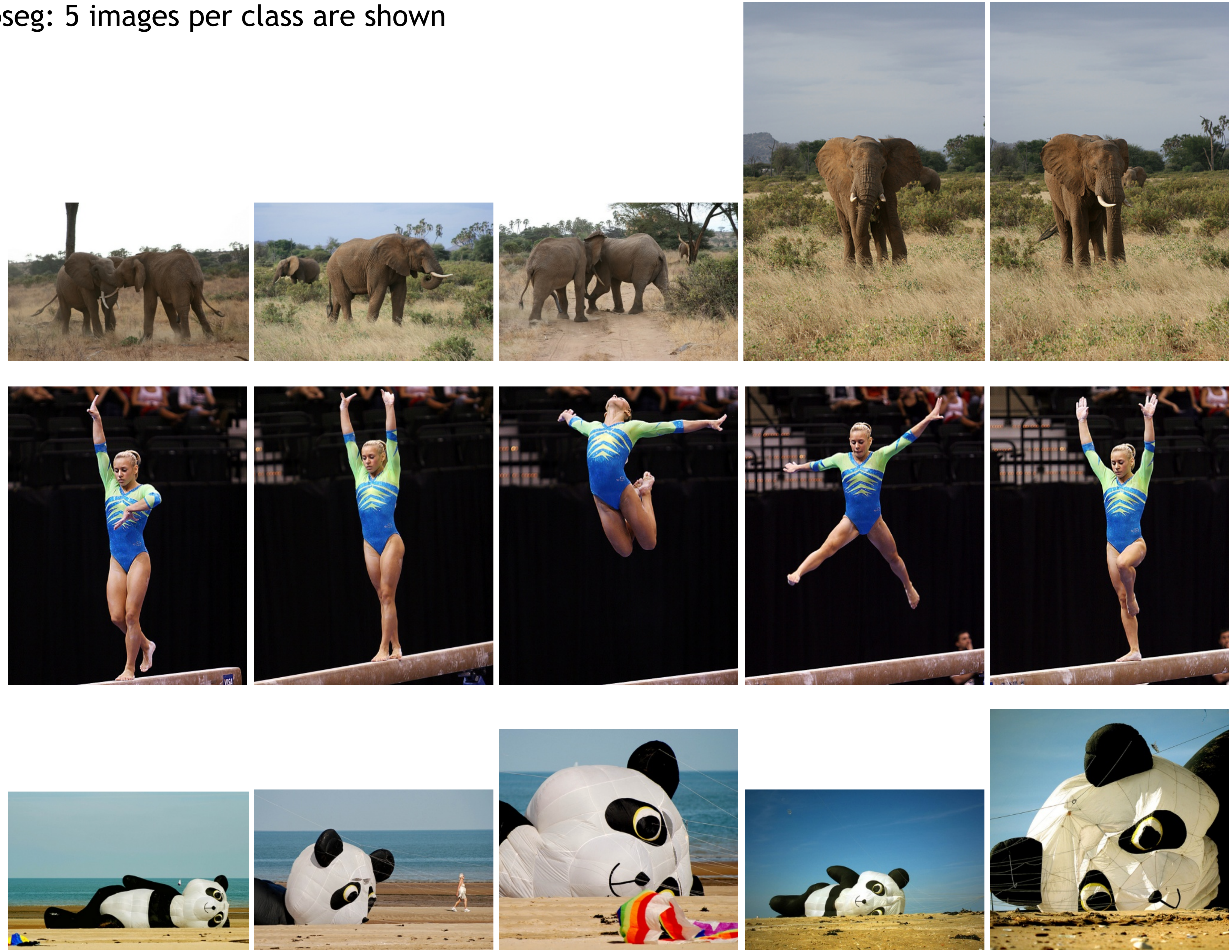
iCoseg: 5 images per class are shown



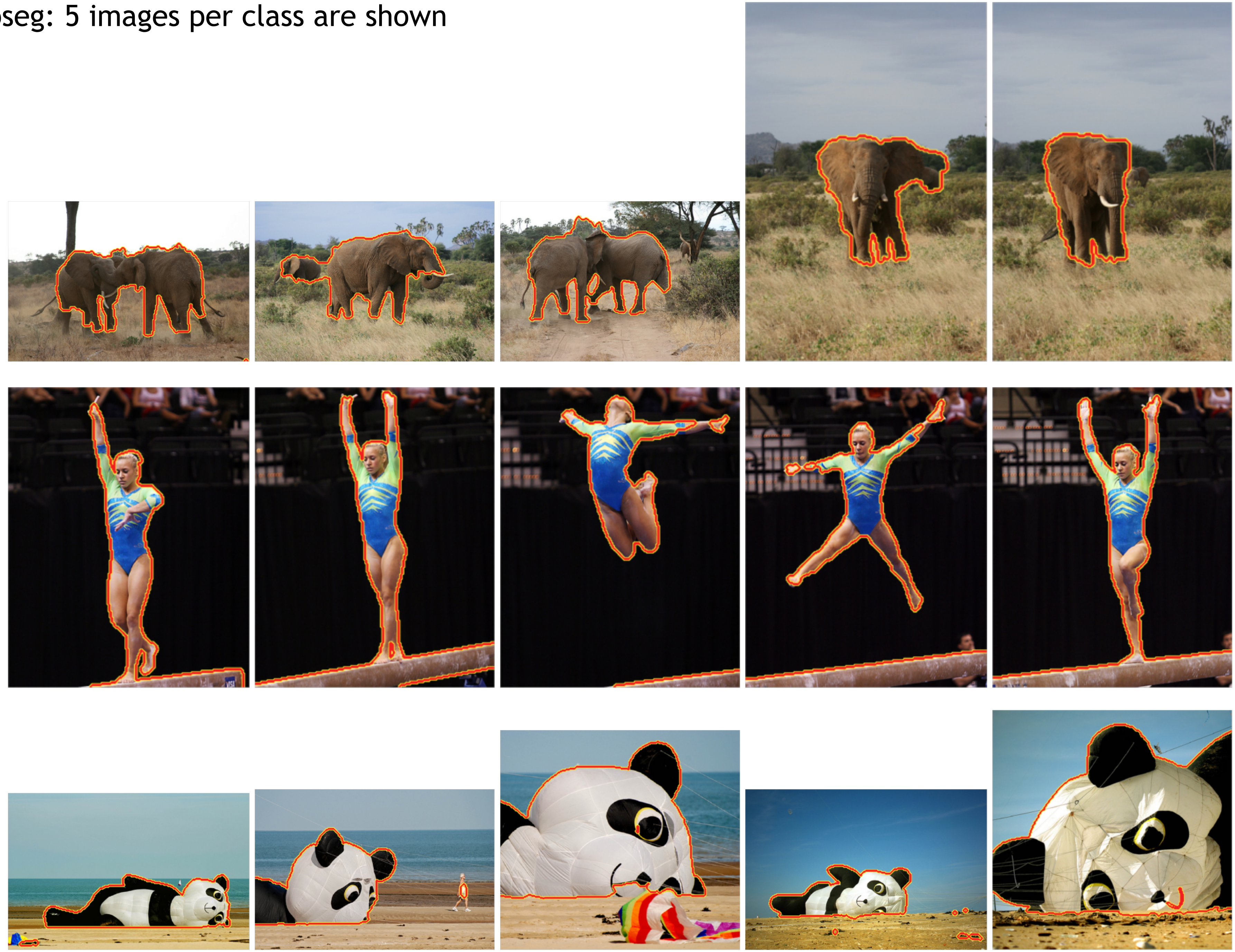
iCoseg: 5 images per class are shown



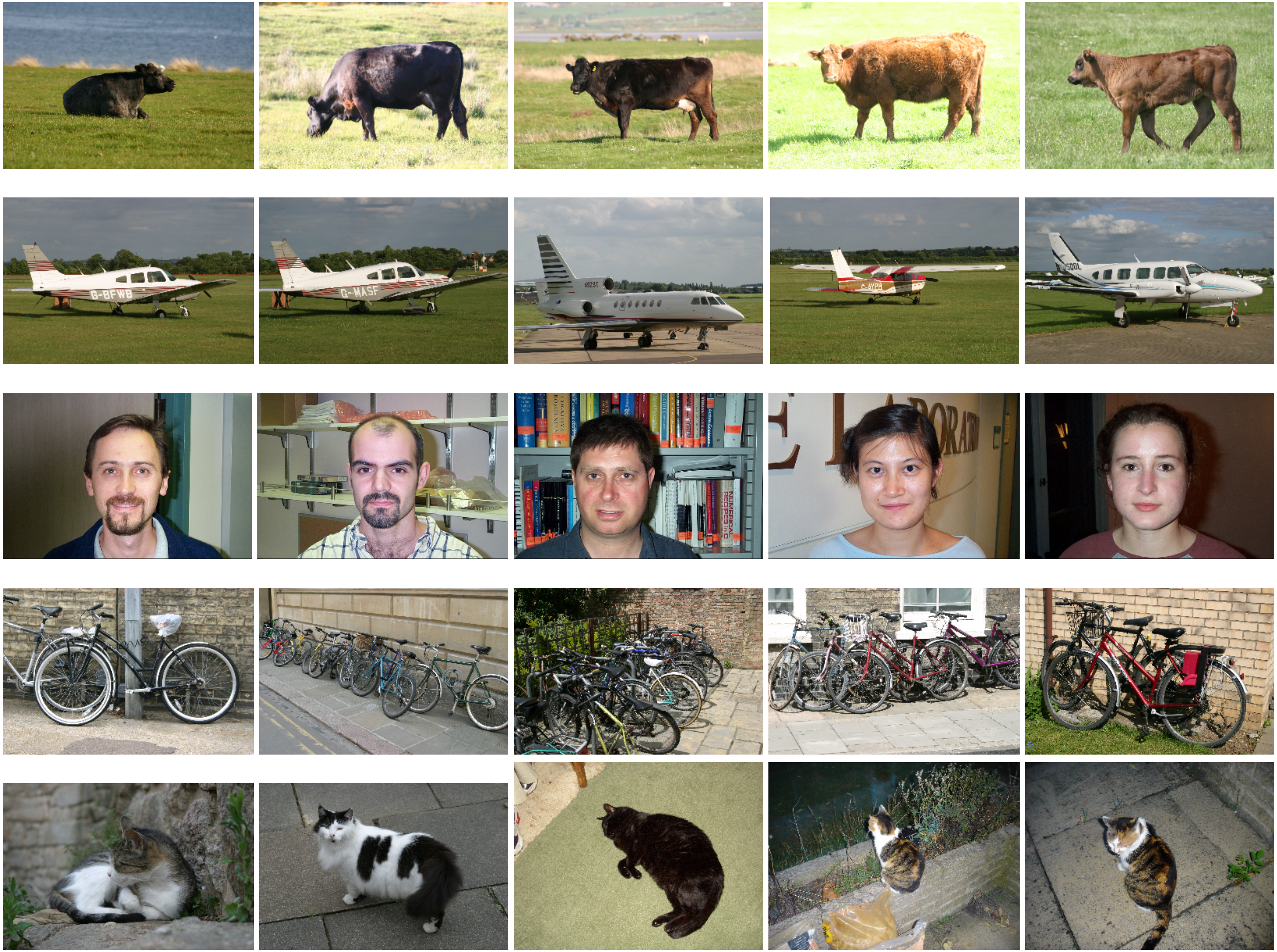
iCoseg: 5 images per class are shown



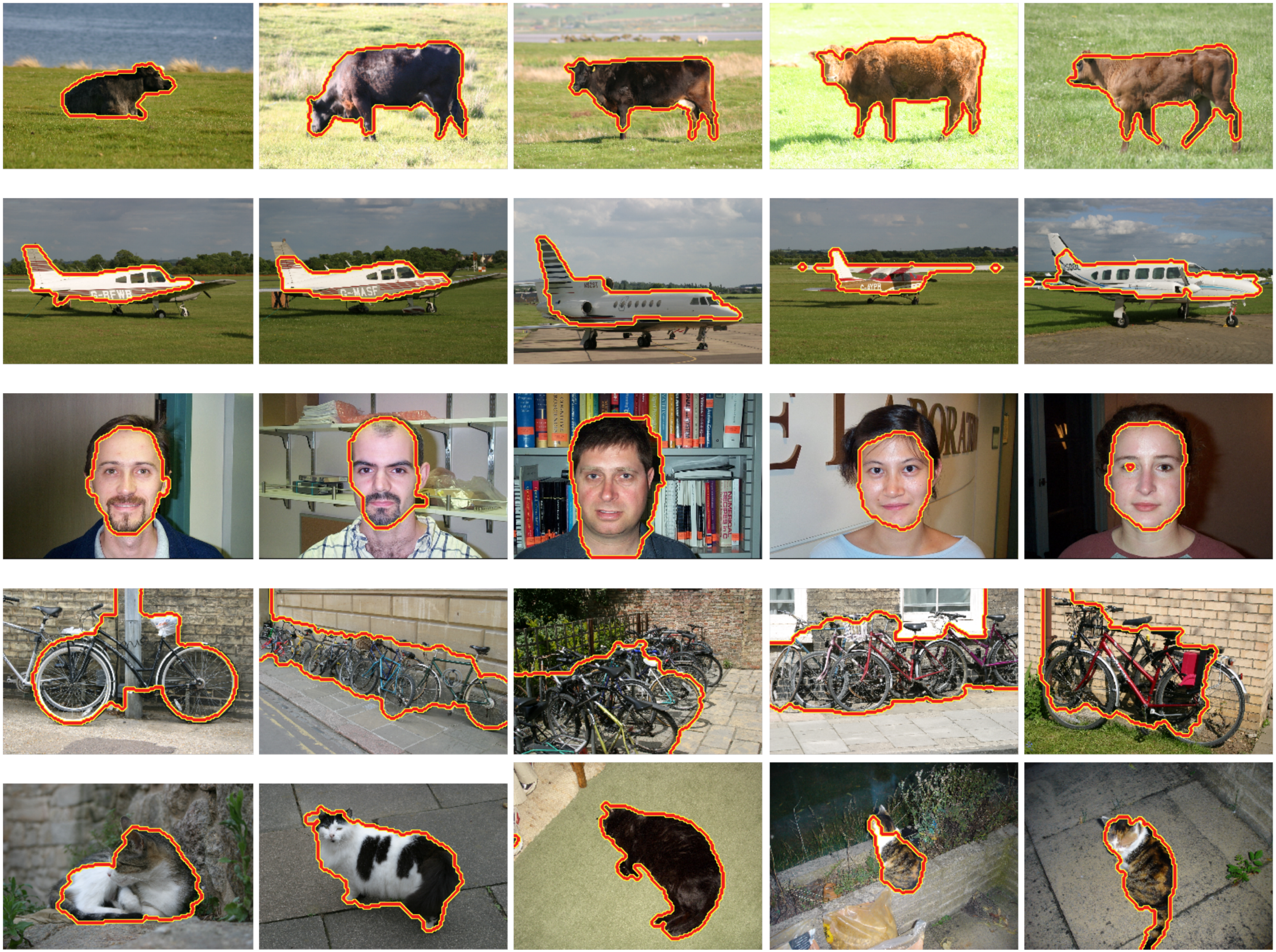
iCoseg: 5 images per class are shown



MSRC: 5 images per class are shown



MSRC: 5 images per class are shown



PASCAL: 10 images per class are shown

PASCAL: 10 images per class are shown

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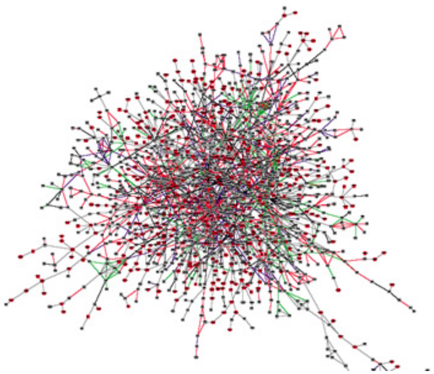
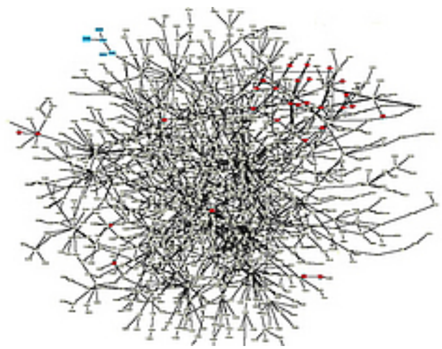
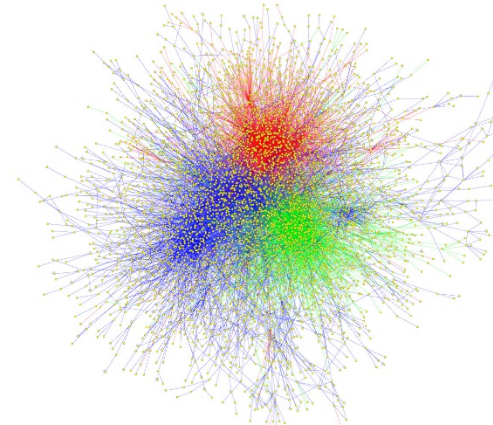
Aligning multiple protein-protein interaction networks



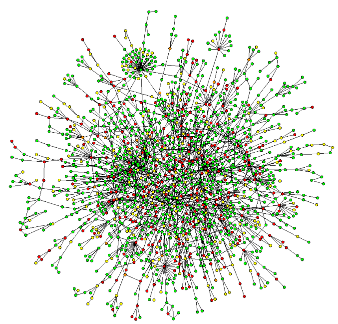
Mouse
(2897 nodes)



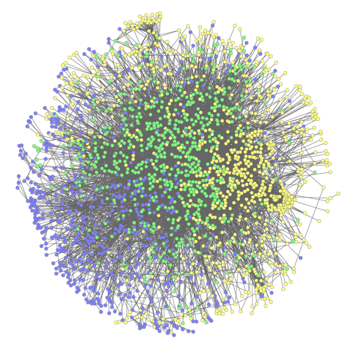
Yeast
(5674 nodes)



Human
(9006 nodes)



Celeg
(4305 nodes)



Fly
(8374 nodes)

S. Hashemifar, Q. H, J. Xu, ConvexAlign: Joint alignment of multiple networks via convex relaxation, under review

Consistent and simultaneous pair-wise alignments

Protein similarity

Interaction preservation

$$
\begin{aligned}
\text{maximize} \quad & \sum_{1 \le i < j \le N} \left(\langle C_{ij}, X_{ij} \rangle + \lambda_2 \langle \mathbf{1}, \mathbf{y}_{ij} \rangle \right) \\
\text{subject to} \quad & \boldsymbol{y}_{ij} \ge \mathbf{0},\ B_{ij}\boldsymbol{y}_{ij} \le \mathcal{F}_{ij}(X), \qquad 1 \le i < j \le N \\
& X_{ij}\mathbf{1} \le \mathbf{1},\ X_{ij}^T\mathbf{1} \le \mathbf{1},\ X_{ij} \ge 0, \qquad 1 \le i < j \le N \\
& X \succeq 0, \quad X_{ii} = I_{|V_i|}, \qquad 1 \le i \le N
\end{aligned}
$$

S. Hashemifar, Q. H, J. Xu, ConvexAlign: Joint alignment of multiple networks via convex relaxation, under review

Biologically more accurate than existing methods

		IsoRankN	SMETANA	NetCoffee	BEAMS	ConvexAlign
c=3	consistent	231	188	462	1084	**1155**
	annotated	2210	1556	1640	2441	**1741**
	specifity	0.10	0.12	0.28	0.44	**0.66**
c=4	consistent	54	170	406	606	**661**
	annotated	942	2019	1640	1138	**1079**
	specifity	0.06	0.08	0.25	0.55	**0.61**
c=5	consistent	9	183	406	359	**493**
	annotated	184	1621	1955	600	**763**
	specifity	0.05	0.11	0.21	0.60	**0.65**
$c \geq 2$	specifity	0.17	0.14	0.29	0.48	**0.71**
COI		127	480	553	1305	**1668**
COI/CI		0.03	0.04	0.21	0.41	**0.59**
Sensitivity		0.45	0.36	0.22	0.37	**0.51**

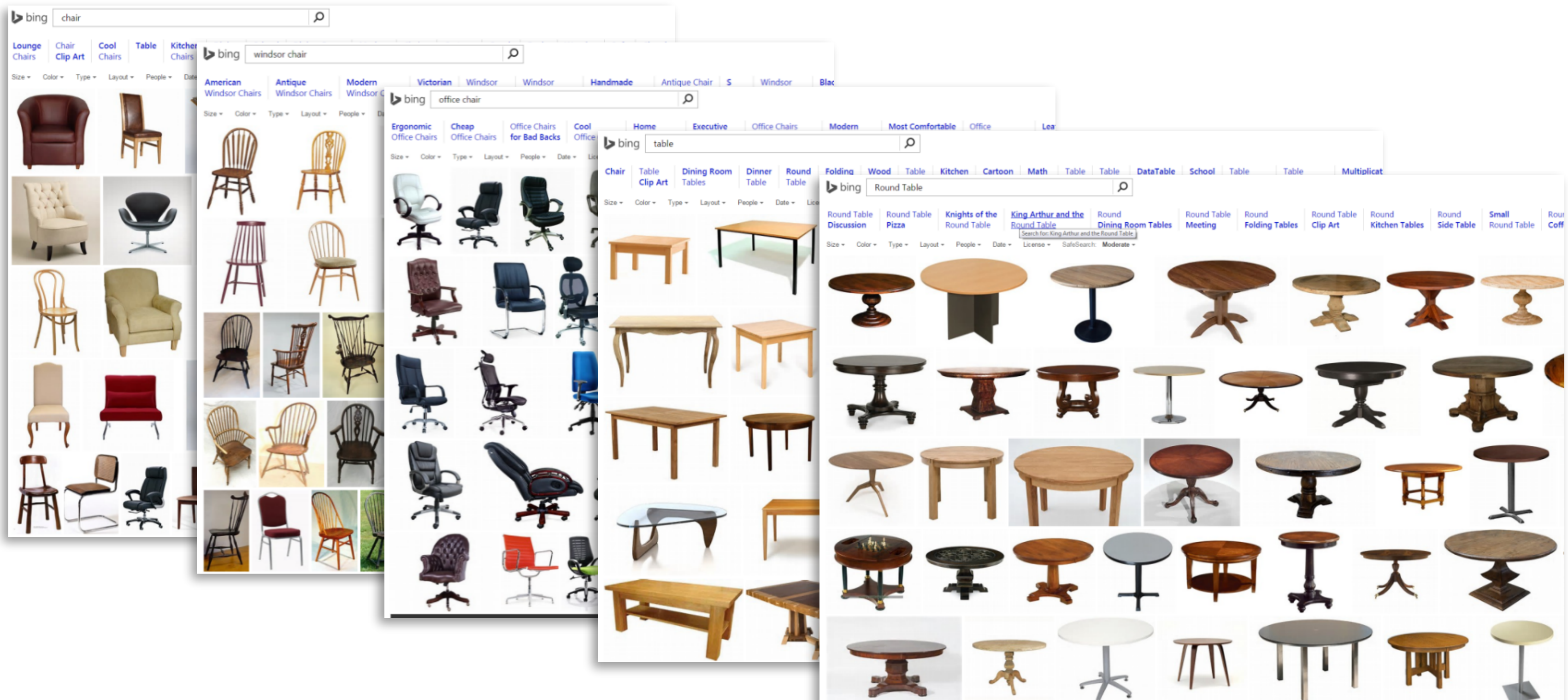
Concluding remarks

Discussion

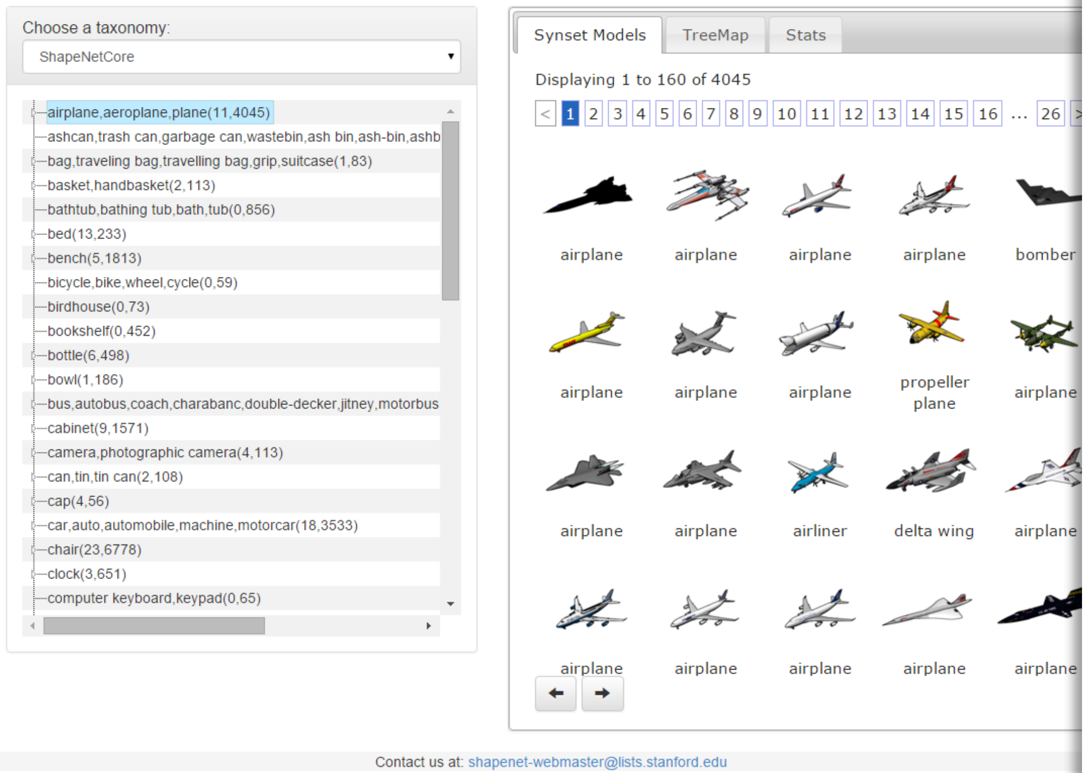
- Map synchronization is generalized from
  - Nearest-neighbor
  - Graph-based semi-supervised learning

- Cycle-consistency provides strong regularization for maps

Organized object collections are becoming more and more accessible



Organized object collections are becoming more and more accessible



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- Shuran Song (Princeton)
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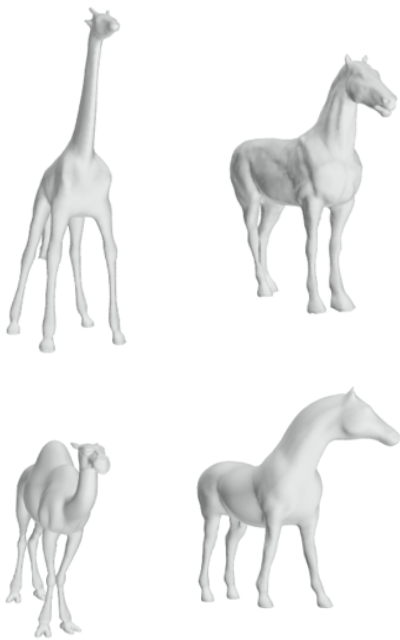
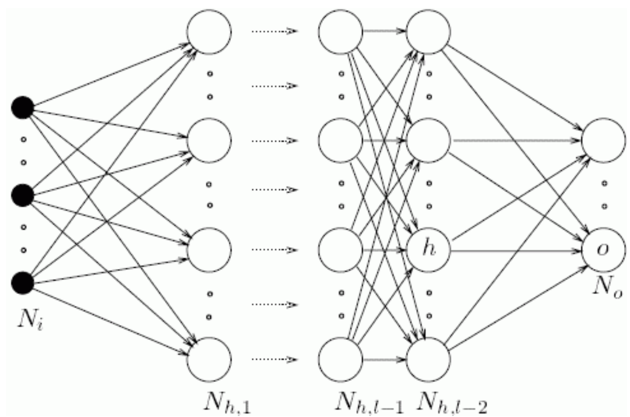
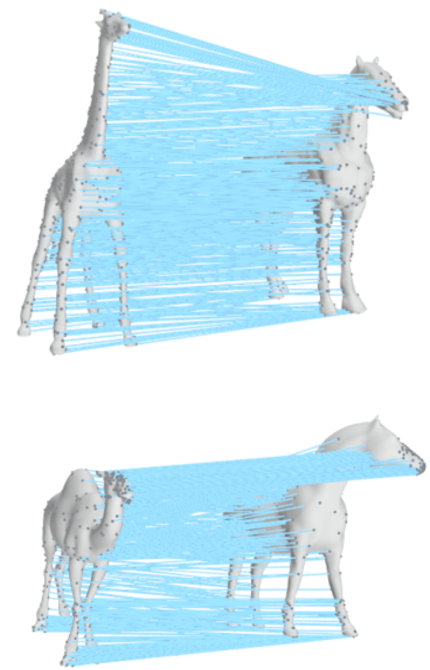
- Zhoutong Zhang (Tsinghua/Stanford)

ShapeNet

Learning object matching

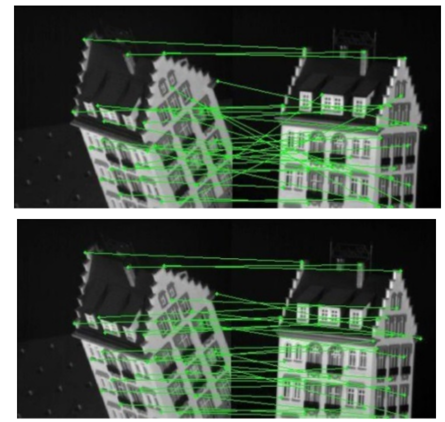






Requires large-scale labeled training data

Map synchronization versus learning pair-wise matching



CMU Hotel dataset

Pair-wise (RANSAC)	Pairwise (Learning) Leordeanu et al. 12	Joint Matching (from RANSAC)
64.1%	94.8%	99.9%

Thank you!